901. Substitute to find a quadratic equation. Consider the discriminant algebraically, then describe the result in terms of the graphs.

902. The natural logarithm is defined as $\ln x \equiv \log_e x$. Translated, this is "whatever you need to raise e by to get x".

903. Finding "the best linear approximation" is the same process as finding a tangent line.

904. In this problem, the square is the possibility space, and area corresponds to probability.

905. In e.g. (a), sketch $x+y=k$. Note that correlation is closeness to a *linear* relationship, not the mere existence of a relationship.

906. For a quadratic sequence $Q_n = an^2 + bn + c$, the second difference is $2a$.

907. Draw gradient triangles.

908. (a) Differentiate with respect to y.
  (b) Use $y - y_1 = m(x - x_1)$.

909. If two factors multiply to give zero, then...

910. Let a be the length of the diagonal which is a line of symmetry, and b the perpendicular diagonal. See the kite as two symmetrical triangles back to back, each with base a and perpendicular height $\frac{1}{2}b$.

911. For a double root, you need a squared factor. The quadratic factor has two single roots. Test these in the cubic factor.

912. Consider common factors in the numerator and denominator.

913. The interior angles must sum to 2π. Consider the average value of the angles.

914. (a) Set up $r = kT^{\frac{2}{3}}$ and substitute to find k.
  (b) Substitute $T = 2$.

915. Rewrite the logarithmic equation as its equivalent index equation. Then take the positive square root of both sides. Use an index law to simplify.

916. You do not need to find the coordinates of the points explicitly to prove the required result. You can use symmetry, if you identify and justify the symmetry clearly.

917. (a) The domain is the set of inputs.
  (b) The codomain is a set which is guaranteed to contain the outputs. It is often larger than the range: for instance, $\mathbb{R}$ is a possible codomain for $x \mapsto x^2$, whose outputs are positive.
  (c) The range is the exact set of inputs attainable with the given domain, e.g. $\mathbb{R}^+$ for $x \mapsto x^2$.

918. Cube both sides to give $y^3 = x$. To sketch this, sketch first the graph $y = x^3$. Then reflect in the line $y = x$, which corresponds to switching the roles of x and y.

919. In a combinatorial approach, i.e. counting equally likely outcomes, visualise the possibility space, drawing it if necessary. Then use $p = \frac{\text{successful}}{\text{total}}$.

— Alternative Method —

In a conditioning approach, i.e. working trial by trial, consider rolling the four-sided die first.

920. Use the binomial expansion (only once) to simplify the numerator. Half of the terms will cancel. You should find a common factor of 2^x.

921. Set up the equation for intersections, which is a polynomial. Consider the degree of this equation, and the fact that a polynomial of degree k has at most k roots.

922. Write down three equations, each for an aspect of equilibrium: horizontal, vertical and moments around e.g. the bottom-right corner. Solve these for F, G and H.

923. Differentiate and set the derivative to 456. Solve the resulting quadratic equation and identify the relevant root by testing it in both the line and the curve. Alternatively, show that the equation for intersections has a double root.

924. Having squared the equations, add them. Then use the first Pythagorean trig identity.

925. Use the vector **suva**t $\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$ to calculate the displacement vector. Then divide this by the duration of the interval.

926. Euler's number e, whose main role is as the base of the standard exponential $x \mapsto e^x$, is around 2.7. The first graph is standard exponential growth. Consider the second graph as a transformation of the first: each minus sign represents reflection.

927. It makes no difference where the 1 goes. So, place it, and then consider the possible arrangements of $2, 3, 4$.

928. Rearrange the circle equation into the form

$$(x-a)^2+(y-b)^2=r^2.$$

Find the intersection of the lines, and evaluate the LHS of the circle equation at this point. Compare the result to your r^2.

929. The symmetries are called "even" and "odd". You might consider the curves $y=x^2$ and $y=x^3$, which have these symmetries.

930. This is a quadratic in a^2b.

931. The key fact is "with replacement".

932. Calculate both quantities in terms of the side length l, and then substitute for l.

933. Consider the situation after 1 hour, and show that it represents the situation at any time.

934. Consider the symmetry of the octahedron.

935. Begin with $3(t-1)^2$, then deal with the term in x.

— Alternative Method —

Let $x=t-1$, then substitute $t=x+1$.

936. This is true.

937. Sketch the curve before setting up the appropriate definite integral.

938. Consider the integral of $\text{f}'(x)$: in particular, the constant of integration.

939. Consider the probability that each tile ends up with a shaded edge on the common border.

940. The answer is "Yes" for two, "No" for one.

941. Work in degrees first if necessary.

942. Take out a common factor first.

943. Remember that f^{-1} is the inverse of f, and f^2 is the function applied twice.

944. Consider the validity of the first sentence.

945. "At least" refers to the fact that the polygons might overlap. To analyse the boundary case, in which the remaining angle is minimal, assume that they do not.

946. (a) The integrand is velocity; its derivative is the acceleration. The duration of the motion is the difference between the time limits.
(b) The integrand is velocity. Evaluate it at $t=2$ and $t=6$.
(c) Perform the definite integral.
(d) Use $s=ut+\frac{1}{2}at^2$.

947. Either consider the vector displacement when $\Delta\lambda=1$, or find the endpoints explicitly.

948. To disprove this, quote the assumptions necessary for the use of a binomial distribution. There are many reasons why it is incorrect: you only need one of them.

949. For equilibrium, the resultant force in both x and y must be zero. So, set up simultaneous equations.

950. Sketch the boundary equations, with dashed lines for strict inequality, and then shade regions.

951. Two are true, one is false. Remember that a square is every type of quadrilateral.

952. The notation is unfamiliar, but just substitute for f and carry out the integral to set up a standard equation in a.

953. Differentiate to find the gradient formula $\frac{dy}{dx}$. Sub in $x=1$ to find the gradient at this point. Hence, show that the equation the tangent is $3x+2y=k$. Find the y coordinate at $x=1$. Hence, find k.

954. Write each set in interval set notation, then it will be easier to visualise the intersection. The latter set is all x values which differ from -2 by more than 2.

955. (a) Consider the equation for intersections.
(b) Consider the equation for intersections.
(c) Sketch the graphs. You might want to consider the cases $x<1$ and $x\geq 1$ separately.

956. With your calculator set in radians, use the cosine rule. Note that the largest angle is always opposite the longest side.

957. The possibility space is originally {HH, HT, TH, TT}. Restrict this using the condition given. Then find the probability, in the restricted possibility space, using $p=\frac{\text{successful}}{\text{total}}$.

958. (a) $x=(y-a)(y-b)$ is a positive parabola with y intercepts at $y=a$ and $y=b$.

(b) $x = (y-a)^2(y-b)$ is a positive cubic with a double root (just touches) at $y = a$ and a single root (crosses) at $y = b$.

959. Set up and solve simultaneous equations.

960. Use circle geometry, rather than calculus.

961. Find and simplify an algebraic expression for the first difference $d_n = u_{n+1} - u_n$. Show that the d_n is the formula for an AP and give its common difference.

962. The fixed points of a function g satisfy $\mathrm{g}(x) = x$. Set up and solve a quadratic equation.

963. In each case, the answer corresponds to a standard exact trigonometric value.

964. In both (a) and (b), consider gradient triangles.

965. The roots of the boundary equation must be $x = 4$ and $x = 5$. Use the factor theorem.

966. Use the factorial definition ${}^n\mathrm{C}_r = \dfrac{n!}{r!(n-r)!}$.

967. Consider the behaviour of a curve at double and triple roots. The relevant fact is whether the curve *crosses* the x axis, i.e. whether there is a sign change in y.

968. There is no calculation required here. $\mathbb{P}(A \mid B)$ means "The probability of event A occurring, given the fact that event B is known to have occurred."

969. Consider a quadratic equation in x^3.

970. Consider transformations of the graph $y = \mathrm{f}(x)$. All of the transformations involved are output transformations.

971. Expand the numerator binomially, and simplify.

972. (a) Differentiate.
     (b) Substitute the point $(a, {}^1\!/\!_a)$.

973. Write the sum out longhand. You should get a quadratic in x.

974. This is equivalent to solving simultaneously for a and b: use elimination or substitution.

975. Ending in 0 is equivalent to having a factor of 10, which is equivalent to having a factor of 2 and a factor of 5.

976. ABC is a right-angled triangle. Use Pythagoras to find its lengths, then trig for the angle.

977. The numerator is a quadratic. So, for the fraction to be expressible as a linear function of x, $x - 6$ must be a factor of the numerator.

978. The boat will go backwards. Explain why.

979. If you can, sketch directly as a transformation of $x = \sqrt{y}$. Otherwise, you can square both sides, but remember that this can (and in this case, does) produce extra (x, y) solution points. The original graph $\sqrt{y} = x - 1$ is half of a parabola.

980. Draw the possibility space (6×6 grid) and count outcomes.

981. Translate the information in the first sentence into algebra, using a variable x and constants $a \neq 0$, b and c in the quadratic. Then differentiate both sides with respect to x.

982. This is true. Each side is an equivalent definition of independence. To prove the implication, use the formula $\mathbb{P}(A \cap B) = \mathbb{P}(A \mid B)\,\mathbb{P}(B)$.

983. Consider any values where the denominator is zero, with reference to the domain $[0, 1]$.

984. Express this algebraically, with first term a and third term $a + 8$. Solve the resulting quadratic.

985. Simplify $(2n+1)^2 - (2n-1)^2$

986. (a) Solve the equation for intersections.
     (b) Consider the y distance between the curves.
     (c) Carry out the integral.

987. In both (a) and (b), give the results in terms of the binomial coefficients ${}^n\mathrm{C}_r$. The number of ways of choosing r squared from n is ${}^n\mathrm{C}_r$.

988. The gradients of the vectors must multiply to -1. Write this algebraically and rearrange.

989. The input transformation is replacement of x by $2x$. The output transformation is multiplication by 2.

990. The equations represent distinct parallel lines.

991. In each of the four bounds, k appears in the same fashion: changing k translates the intervals along the number line. So, you can set $k = 0$ and solve the problem, then reintroduce k afterwards.

992. Numbers $1, 2, 5, 6$ may be placed without loss of generality, as any placement may be rotated into any other. This leaves two opposite faces, for 3 and 4. Give an explicit configuration that is different from the one shown in the question.

993. Use the cosine rule to find the cosine of an angle. Convert this into a sine with the first Pythagorean trig identity. Then use $A_{\triangle} = \frac{1}{2}ab\sin C$.

——— Alternative Method ———

Use Heron's formula, in terms of semiperimeter s:

$$A = \sqrt{s(s-a)(s-b)(s-c)}.$$

994. Draw a force diagram with the packing case as a particle. Include the angle 7° between the weight and the normal to the slope. Since the friction is dynamic, use the maximal value $F_{\text{max}} = \mu R$. Resolve along the slope.

995. Find the point (p, q) first, by completing the square. Alternatively, reflect the parabola in the y axis by replacing x by $-x$.

996. You can treat the y values as a sequence, since x increases linearly: a quadratic relationship must have a constant second difference.

——— Alternative Method ———

Set up the relationship $y = ax^2 + bx$.

997. Use the fact that there are $n!$ orders of n different objects. There isn't a standard inverse factorial function, so you need to test values.

998. (a) Multiply by the denominator of the LHS.
 (b) Equate coefficients.
 (c) Consider the repeated factor x^2.

999. These are a GP and an AP respectively.

1000. This is a quadratic in $x^{0.2}$.

——— End of Volume I ———